1. **N supercomputers, add edges to create a connected graph  
   Each node gains revenue of C (constant) when the graph is connected  
   Each node pays the cost E (constant) of adding an edge to another node**

**What are the Nash Equilibrium subgraphs?**  
Without adding more nodes, revenue is maximized by minimizing cost.  
All solutions with minimum cost will have N-1 edges. For instance, one of these solutions has one node as a central node and create an edge between each other node and the closest surrounding node up to N-1.

**What is a Nash Equilibrium if edges have non-uniform costs? Find a solution with minimum total cost (sum of all edge costs E)?**   
All solutions with minimum cost still have N-1 edges, but the edges that are selected are now much more important. A proper solution to this problem can be created by trying all possible edges in ascending order of cost and keeping these edges if they connect two nodes that previously had no math to one another.

1. **Find all pure Nash Equilibria of a sealed bid first-price auction. Does player 1 have a special place?**  
   The player with the highest valuation can always win because they can outbid the others, so this player does have a special place in this game; your standing in the game depends entirely on your valuation. Equilibrium would occur anytime that player 1 has placed the minimum bid to beat all other players, satisfying (v1>=b1=(v2+1)) > (v2>=b2) >= (v3>=b3) … >= (vn>=bn). No other players would have any incentive to change their bid, as they can’t exceed their valuation and decreasing their bid has no effect.

**Find all possible Nash Equilibria for a 3-player second-price auction with valuations v1, v2, v3.**  
Assuming v1 > v2 > v3, v1 will always bet above the value of v2 because it can always win. If v1 bets above v2, v2 bets their full value to increase the price for v1. The bet of v3 doesn’t have any effect on the outcome; they cannot win, and they cannot increase the price for others, so they will not have any incentive to change their bet no matter what it is. Therefore, any case in (v1>=b1) > (v2=b2) >= (v3>=b3) would satisfy Nash Equilibrium. The biggest difference from the first-price auction is that the actual bid of player 1 can be as high as they want as long as the second bid is less than or equal to their valuation.

If all participants were unaware of the valuations of the others, the winning strategy would be to make a bet equal to their valuation. If the player ended up being the first or second highest valuation bidding, they would either win the item being auctioned or succeed in driving up the price for the other bidder.

1. **N consumers assigned to P providers. They get rate Ri = B / Sum(j) of Lj where Lj is the network load of the jth consumer and B is the bandwidth. Payoff is rate offered.**  
   **Find Nash Eq when everyone has the same load.**  
   Assuming equal bandwidth between ISPs, the rate is inversely dependent on the load, i.e. more load means lower rates, therefore customers will go for the ISP with the least load. Assuming also that consumers make their decisions based on what the rate would be if they joined the service provider and not the rate before they switch, the Nash equilibrium is the N consumers divided as equally as possible between providers (e.g. 9 consumers across 3 providers would yield 3 consumers with each provider).

**Show that a Nash Eq exists when loads are different**  
I will use an example to show this is true. Suppose there are 8 consumers and 2 service providers. The loads are as follows: Lj = [ 4, 2, 8, 2, 3, 4, 1, 5 ].

If the first four are in ISP1 and the last four are in ISP2, the loads are 16 and 13 respectively. So if we look at consumers 2 and 4, they each would have incentive to switch to ISP2, which would give them a higher rate since they only have a load of 2. Let’s say consumer 2 switches. Now the loads are 14 and 15 respectively, with 3 consumers in ISP1 and 5 consumers in ISP2. Although the 7th consumer could switch to ISP1 with no ill effects, they have no incentive to do so as the rate would be the same, so this is our Nash Equilibrium with the load listed for each consumer:

ISP1 = [ c1=4, c3=8, c4=2 ] ISP2 = [ c2=2, c5=3, c6=4, c7=1, c8=5 ]

1. **Determine NE for Tragedy of Commons/Bandwidth with N players where the utility function is: Ui = Xi (K – a SUM{Xj, j}) where Xi is the units of traffic sent by player i.**Because each player seeks to maximize their utility, they seek to maximize their resource usage which consequently minimizes the amount left for other players. Players will increase their Xi as much as possible as quickly as possible until the cap K is hit. No player has any incentive to release their traffic given this utility function, so any situation where players have depleted the resource and hold onto their traffic can be considered a Nash Equilibrium.
2. **Determine NE in a Cournot Game with 2 players where the cost function for producers is:  
   ci = ln (xi) where the utility is defined by Ui = xi\*p(x1 + x2) – ci and p(y) = max{0,K-y} where K is a constant number**Ui = xi\*(K-(x1+x2)) – ci   
   b1 = (max{K – x2,0} – ln(x1))/2   
   b2 = (max{K – x1,0} – ln(x2))/2

There is a Nash Equilibrium where the output of each producer is:  
x1 = (K – (K – x1 – ln(x1))/2) – ln(x1))/2  
x2 = (K – (K – x2 – ln(x2))/2) – ln(x2))/2